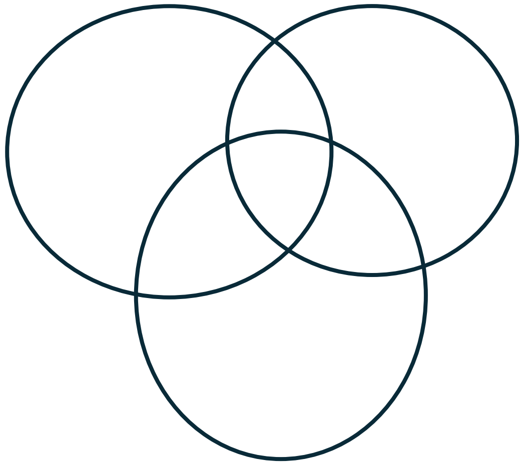

The Bell Inequality

An easy approach using a Venn Diagram

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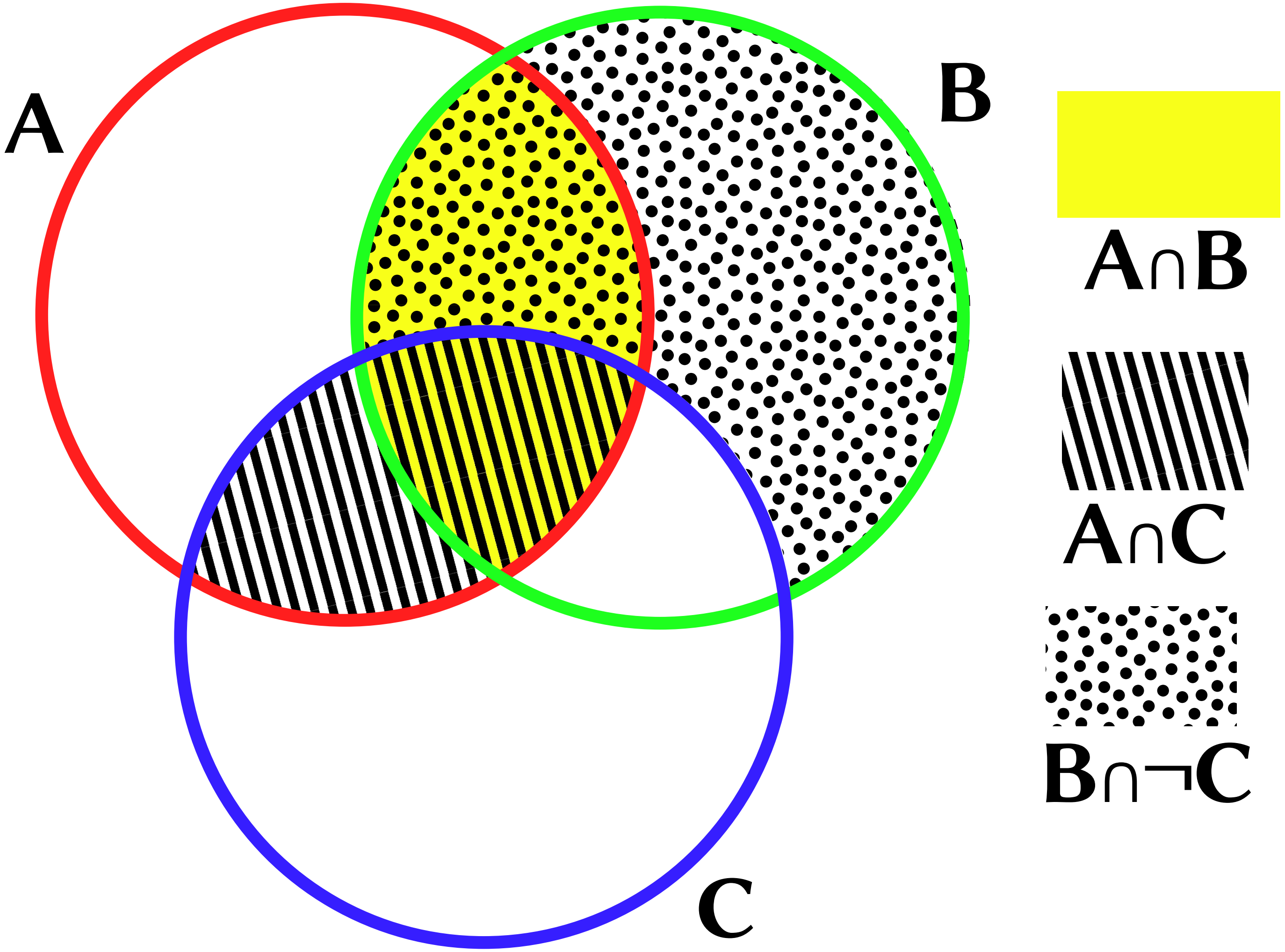
A Bell Inequality

Note there are two properties per term in the inequality

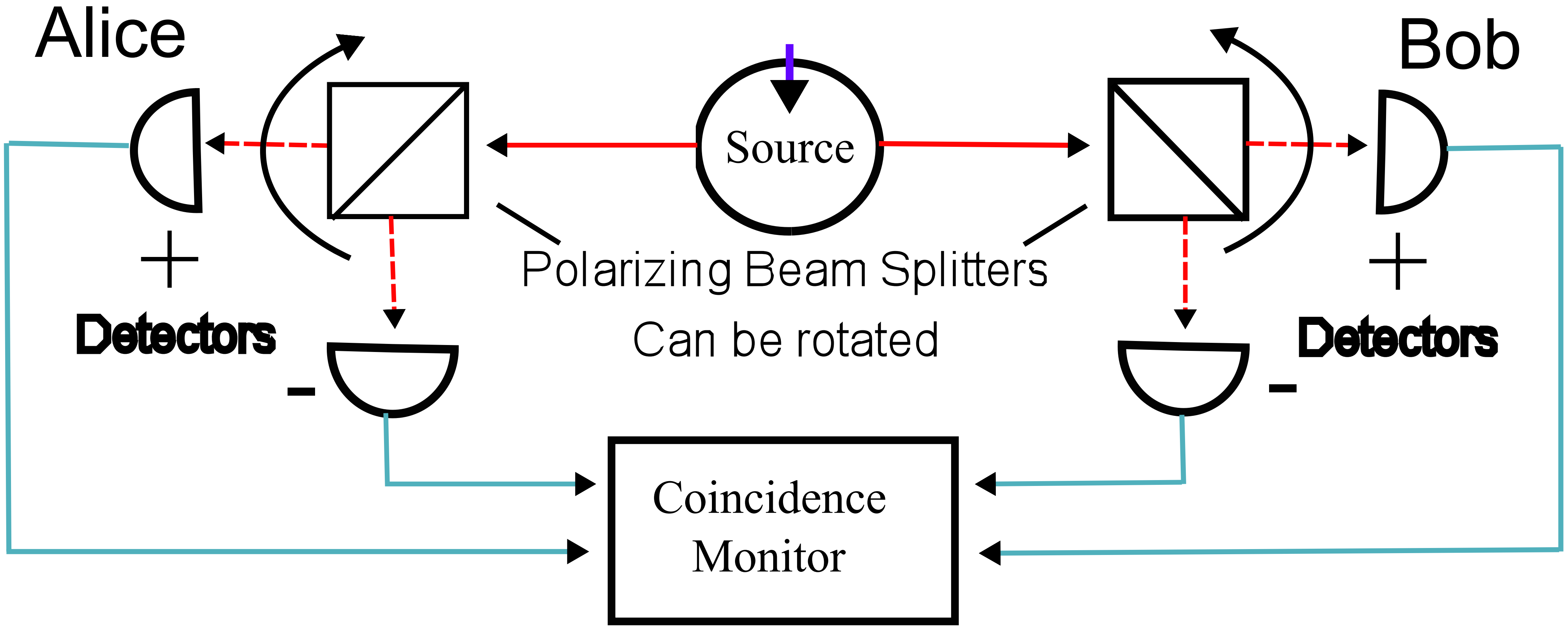
The inequality is pretty obviously true where properties A, B, and C can be measured for each thing in the sets.

A Bell Inequality as there are other inequalities also used in Bell tests. This one is due to Eugene Wigner.



$$|A \cap B| \leq |A \cap C| + |B \cap \neg C|$$

Bell test with polarised light



Source can use SPDC (spontaneous parametric down-conversion)
for instance to generate pairs of entagled photons

Light through a polarizing filter

- If light is passed through a polarizing filter then any light that comes out is polarized.
- The amount that passes through a second polarizing filter is $\cos^2\theta$ where θ is the difference in the polarizing angle of the two filters.
- For entangled photons going through the Bell test if the polarizing beam splitters are at an angle of θ relative to each other, then if a photon is detected going through a detector on one side it has a chance of $\cos^2\theta$ of going through the corresponding detector on the other side, just like the original photon would do with its polarization.

Hidden variables?

- If the angle difference is 0° all the light will pass through the same detectors at both ends, either both + or both -.
- The easiest theory to account for this is that the photons started off with a hidden state so they know for every angle what result they should give.
- That means that whether a photon would go through detector + or - is fixed for every angle when it leaves the source and tests with different angles should satisfy Bell’s Inequality.

A Bell Test – Spooky Action at a Distance

- Alice and Bob turn their polarizing Beam Splitters at random to angles 0°, 30°, and 60°. What do they get?

- Property A is if one has angle 0° and they detect +.
- Property B is the same for 30° and property C for 60°
- Then $|\mathbf{A} \cap \mathbf{B}| = \cos^2 30° = 3/4$, $|\mathbf{A} \cap \mathbf{C}| = \cos^2 60° = 1/4$
- $|\mathbf{B} \cap \neg\mathbf{C}| = 1 - \cos^2 30° = 1 - 3/4 = 1/4$

- But $3/4 \leq 1/4 + 1/4$ is false! - Bell’s Inequality is violated!